

STUDYING THE STREAMLINING OF A BLUNTED BODY  
 BY A VIBRATIONAL-NONEQUILIBRIUM DISSOCIATED GAS

O. Yu. Apolonskii, P. E. Babikov, I. V. Lebed',  
 and V. V. Ryabov

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It is usual in the study of the streamlining of bodies by a nonequilibrium flow of dissociated air to assume equilibrium in the translational, rotational, and vibrational degrees of freedom. However, this approximation does not offer the required accuracy, for example, in examining the fast-acting processes that arise in compression shocks. To describe the structure of the flow in this region it thus becomes necessary to make provision for nonequilibrium vibrational relaxation.

In the present study we solve the problem of the streamlining of a blunted body by a vibrational-nonequilibrium dissociated gas within the framework of the CVDV model [1, 2] which takes into consideration the relationship between dissociation and the vibrations. Particular attention is devoted to an analysis of the effect that the processes of dissociation and vibrational nonequilibrium have on the thermal loads of a flight vehicle.

We have to make a distinction between two typical situations: the streamlining of a body under conditions of free natural flight and the streamlining of a model in an aerodynamic tunnel (AT). If the approaching stream in the first case is in a state of equilibrium, then under the conditions of the tunnel experiment it is vibrationally "frozen in" and exhibits a level of dissociation different from zero [3]. In this particular study we examine the possible methods of simulating the natural conditions of flight in aerodynamic wind tunnels.

The progress of the processes of dissociation and vibrational nonequilibrium in the presence of moderate Reynolds numbers was examined in [4] on the basis of a model of a hypersonic shock layer. The cited theoretical data enabled us to ascertain the significant role played by diffusion near the front of the shock wave.

However, the mean-energy equation for the relaxation of vibrations that was used there failed to take into consideration the presence of an atomic component in the dissociated gas. In the following we take this effect into consideration. Specific calculations have been carried out for oxygen and nitrogen.

1. The Physical Flow Model. The motion of a relaxing gas mixture consisting of atomic and molecular components is described by a system of conservation equations [5]. Let the rotational degrees of molecular freedom be in equilibrium with the translational degrees of freedom, while no such equilibrium exists between the vibrational and translational degrees of freedom. Then the expressions for the rates of diffusion  $V_M$  and  $V_a$ , for the velocity  $q$  of the heat flux, and for the average internal energy of a unit volume of gas  $\langle E \rangle$ , contained within the system of conservation equations, may be brought to the following form [5]:

$$\begin{aligned}
 V_M &= -\frac{n_a^2}{\rho} \frac{m_a}{n_M} D_{am} d_m, \quad n_M m_M V_M + n_a m_a V_a = 0, \\
 q &= -\left( \lambda' + \frac{n_M D_{MM}}{1 + \frac{n_a D_{MM}}{n D_{am}}} k \right) \text{grad } T + \\
 &+ \left[ \frac{7}{2} kT + \langle E_V \rangle - \frac{m_M}{m_a} \left( \frac{5}{2} kT + \frac{D}{2} \right) \right] n_M V_M - \frac{n_M D_{MM}}{1 + \frac{n_a D_{MM}}{n D_{am}}} \text{grad } \langle E_V \rangle, \\
 \langle E \rangle &= (kT + \langle E_V \rangle) n_M + \frac{1}{2} D n_a.
 \end{aligned}$$

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Here  $T$  is the temperature of the gas, determined from the kinetic energy of the particle;  $\langle E_V \rangle$  is the average vibrational energy of a unit volume of gas;  $D$  is the dissociation energy, with the kinetic energy of the molecules calculated from the zero level;  $n$  and  $\rho$  are the concentration and density of the gas;  $m$  is the mass of the particles; the subscripts  $M$  and  $a$  refer to molecules and atoms; the expressions for the coefficients of binary diffusion  $D_{aM}$ , self-diffusion  $D_{MM}$ , thermal conductivity  $\lambda'$ , and viscosity  $\mu$  are presented in [5-7].

The rate of dissociation in the system of conservation equations was calculated within the framework of the CVDV model [1, 2], which takes into consideration the mutual influence of dissociation and vibrational relaxation. Analysis of later studies and comparison of various approximate models with the CVDV model can be found in [5]. According to [1, 2], the correction factor for the dissociation rate constant, brought about by the absence of equilibrium between the vibrational and translational degrees of freedom, has the form

$$k_d = k_d^0 Q, \quad Q_d = Z(T)Z(T_F)/(Z(T_V)N), \quad (1.1)$$

where  $Z$  is the vibrational statistical sum;  $T_V = \langle E_V \rangle/k$ ;  $T_F^{-1} = T_V^{-1} - T^{-1}$ ;  $k_d^0$  is the equilibrium value of  $k_d$  corresponding to  $T_V = T$ ;  $N$  is the whole part of the number  $D/\hbar\omega$  ( $\hbar\omega$  is the magnitude of the vibrational molecular quantum). In the following calculations we use the temperature-related quantities  $k_d^0$  and  $k_r^0$ , constants for the rates of recombination, taken from [8].

To close the system of conservation equations, it has to be enhanced with the equation for  $\langle E_V \rangle$  [6]:

$$\frac{\partial \langle E_V \rangle}{\partial t} + \mathbf{u} \frac{\partial \langle E_V \rangle}{\partial \mathbf{x}} + \mathbf{V}_M \frac{\partial \langle E_V \rangle}{\partial \mathbf{x}} - \frac{1}{n_M} \frac{\partial}{\partial \mathbf{x}} \left[ \frac{n_M D_{MM}}{1 + \frac{n_a D_{MM}}{n D_{aM}}} \frac{\partial \langle E_V \rangle}{\partial \mathbf{x}} \right] = \frac{\langle E_V \rangle^0 - \langle E_V \rangle}{\tau_{V-T}} + \kappa.$$

Here  $\mathbf{u}$  is the velocity vector;  $\langle E_V \rangle^0$  is the equilibrium value of  $\langle E_V \rangle$ ; all of the  $\kappa$  terms from [1, 2] describe the influence of dissociation on vibrational relaxation; the first term in the right-hand side of the equation corresponds to a model of an harmonic oscillator.

In calculating the vibrational relaxation time  $\tau_{V-T}$  we took into consideration that the relaxation of the vibrational energy is achieved in a mixture of atomic and molecular components:  $1/\tau_{V-T} = n_a/n\tau_{M-a} + n_M/n\tau_{M-M}$ .

As is well known, the vibrational relaxation of the  $O_2$  molecule into  $O_2$  and of the  $N_2$  molecule into  $N_2$  and  $N$  is interpreted within the framework of the Landau-Teller adiabatic mechanism, whereas for the description of the vibrational relaxation of  $O_2$  to  $O$  it is necessary to take into consideration the nonadiabatic effect [9]. The temperature relationships which were used in these calculations for the average probabilities (per single gas-kinetic collision) of a single-quantum vibrational transition  $P_{M-i}$  ( $i = a, M$ ) can be found in Fig. 1, where

$$P_{M-i} = \sqrt{\frac{\pi kT}{8} \frac{m_M m_i}{m_M + m_i}} \left[ \sigma_0 \left( 1 - e^{-\frac{\hbar\omega}{kT}} \right) p \tau_{M-i} \right]$$

( $\sigma_0$  is the gas-kinetic cross section of the collision and  $p$  is the pressure). In the calculation of  $P_{M-i}$  we took the data from the experiment described in [10, 11]. Extrapolation of  $P_{M-i}$  to experimentally unattainable regions of temperatures was accomplished by means of the correlational dependence of probability on the adiabatic factor [12].

2. Approximation of a Thin Viscous Shock Layer. To describe the motion of a chemically and vibrational-nonequilibrium gas in the vicinity of the deceleration line, we used one of the approximations of the Navier-Stokes equations, i.e., a model of a thin viscous shock layer (TVSL). A detailed exposition of the formulation of the corresponding boundary-value problem can be found, for example, in [13, 14] and for the sake of brevity this is not presented here.

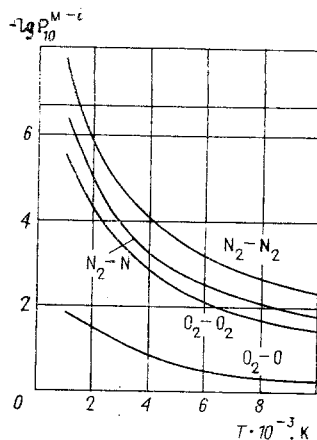


Fig. 1

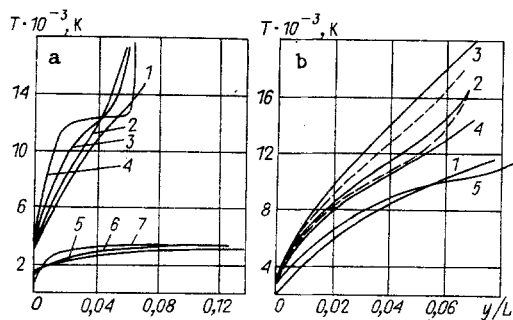


Fig. 2

TABLE 1

Regime	Gas	L, m	$\rho_{\infty} \cdot 10^4, \text{ kg/m}^3$	$V_{\infty}, \text{ m/sec}$	$T_{\infty}, \text{ K}$	$Re_{of}$	$C_h, \text{ catalytic surface}$		St	$C_h, \text{ non-catalytic surface}$	$C_h, \text{ AT}$
							$h_d(T, T_V)$	$h_d^0(T)$			
1	O <sub>2</sub>	0,06	18,46	7900	199	7,08	0,499	0,492	0,548	—	0,318
2	O <sub>2</sub>	0,06	82,83	7900	220	31,8	0,287	0,286	0,317	0,153	0,170
3	O <sub>2</sub>	0,06	309,7	7900	247	118,7	0,135	0,135	0,152	0,069	0,090
4	O <sub>2</sub>	0,06	1030	7900	270,6	575	0,053	0,053	0,062	—	—
5	N <sub>2</sub>	0,1	3,42	7900	187	2,4	0,723	0,722	0,774	—	0,920
6	N <sub>2</sub>	0,1	18,46	7900	199	12,8	0,456	0,452	0,506	—	0,581
7	N <sub>2</sub>	0,1	82,83	6400	220	64,2	0,200	0,199	0,239	—	0,302

In making the quantities contained in the original equation dimensionless, we have taken the following as determining factors: L, the characteristic linear dimension;  $V_{\infty}$ ,  $\rho_{\infty}$ , the velocity and density of the incident flow;  $T_{of}$ ,  $H_0 \approx V_{\infty}^2/2$ , the temperature and enthalpy of deceleration. As a result, the TVSL equations contain a hypersonic approximation parameter  $\varepsilon = (\gamma - 1)/2\gamma$  [13] ( $\gamma$  is the ratio of the specific heat capacities) and the Reynolds number  $Re_{of} = \rho_{\infty} V_{\infty} L / \mu(T_{of})$ .

The conditions of adhesion and nonpenetration are assumed to be specified at the surface of the body, as are the conditions of equilibrium-radiation heat exchange, mass component balance with consideration of various catalytic properties of the surface and, finally, the equilibrium of the vibrational energy. The generalized Rankine-Hugoniot conditions [14] are assumed to be satisfied at the external boundary of the thin viscous shock layer.

For conditions of natural flight we have fulfilled the relationships  $\langle E_V \rangle_{\infty} = \langle E_V \rangle^0(T_{\infty})$ ,  $c_{M,\infty} = 1$ ,  $T = T_{\infty}$  ( $c_{M,\infty}$  is the mass concentration of the molecules in the incident stream). In simulating the conditions for the streamlining of a body in an aerodynamic high-enthalpy wind tunnel, we specified the values of  $\langle E_V \rangle_{\infty}$ ,  $c_{M,\infty}$  and  $T_{\infty}$ , and these are different from the equilibrium values. According to [3], they correspond to the conditions of the "frozen-in" discharge of the dissociated gas from a nozzle.

In the numerical solution of the one-dimensional boundary-value problem, describing the self-similar flow in a nonequilibrium thin shock layer, we made use of a universal procedure for the solution of the systems of nonlinear ordinary algebraic-differential equations with a small parameter for the higher derivatives, in which approximation of the differential equations with respect to the two-point second-order Keller scheme was realized, and we also made use of an original method for the construction of an adaptive computation grid based on minimizing the integral approximation error norm, which is analogous in its idea to the algorithm taken from [15]. In the numerical method which we are considering here, the simultaneous refinement of the grid functions and the distribution of the nodes in the computational grid is accomplished by the Newton-Rafson method, with automatic selection of the iteration parameter, where the factorization of the quasi-free-diagonal Jacobi

matrix is accomplished with monotonic vector sweeping, without any separation of the system of finite-difference equations. The choice of the Keller scheme is based in this case on simplicity and the uniqueness of the approximation of the boundary conditions and the flow parameters contained in the equations, with a joint solution of the difference equations within the framework of a single iteration cycle accomplished to raise the stability of the iterations and to achieve a higher (quadratic) rate of convergence. Using the algorithm of the adaptive grid made it possible effectively to suppress the oscillations of the grid solution which arise with an increase in  $Re_{0f}$  as a consequence of the nonmonotonicity of the discrete approximation, and made it possible to a considerable extent to automate the computational procedure.

Use of the adaptive grids presently serves, apparently, as the only means of achieving high-quality results in calculations of viscous chemically nonequilibrium flows described by moderate and large values for  $Re_{0f}$ .

**3. Calculation Results.** Let us examine the streamlining of a sphere by a flow of a dissociated gas (oxygen or nitrogen) with parameter corresponding to flight at supersonic velocities under the conditions prevailing in seven different regimes (see Table 1). Here we find the values for the heat-transfer coefficient  $C_h = 2q/\rho_\infty V_\infty^3$ , these values significantly dependent on the rarefaction of the medium characterized by  $Re_{0f}$ . The values of  $C_h$  are significantly affected by the degree of catalytic surface activity [8, 14], and with an increase in  $Re_{0f}$  the difference in  $C_h$  for an ideal catalytic surface and one that is absolutely noncatalytic increases (see Table 1).

The distributions of the translational  $T$  and vibrational  $T_v$  temperatures, and of the mass concentration  $c_{O_2}$  of oxygen molecules over the TVSL (for flight regimes 1-4) can be found in Fig. 2a-4a (curves 1-4, respectively). With  $Re_{0f} = 7.08$  and  $31.8$  the flow in the shock layer is completely viscous and the change in the atom concentration at the surface significantly affects the redistribution of the  $O_2$  molecules all the way to the shock wave (Fig. 4a, solid curves represent the ideal catalytic surface, the dashed-dotted curves represent the surface absolutely noncatalytic). With an increase in  $Re_{0f}$  a relaxation layer is clearly formed within the structure of the TVSL near the shock wave (curves 3, 4 in Figs. 2a, 3a), as well as an extensive zone of inviscid flow and a narrow viscous "diffusion" layer at the surface, behind whose boundaries we can assume an extremum in the distribution of the concentration of molecules.

The differences in the distributions of  $T$  and  $T_v$  may lead to a significant difference in the dissociation rate constants  $k_d$  in (1.1) from the "equilibrium" values of  $k_d^0$ . Figures 2a-4a show the values of  $T$ ,  $T_v$ , and  $c_{O_2}$  across the viscous shock layer for the case of non-equilibrium dissociation rate constants  $k_d(T, T_v)$  (the solid curves) and the "equilibrium" values of  $k_d^0(T)$  (the dashed curves). With an increase in the rarefaction of the medium (a reduction in  $Re_{0f}$ ) the influence of this effect on the structure of the flow (particularly on the distribution of the vibrational temperature) increases, and this leads to a reduction in the flow of heat by 2% when using the "equilibrium" values of  $k_d^0$  (see Table 1).

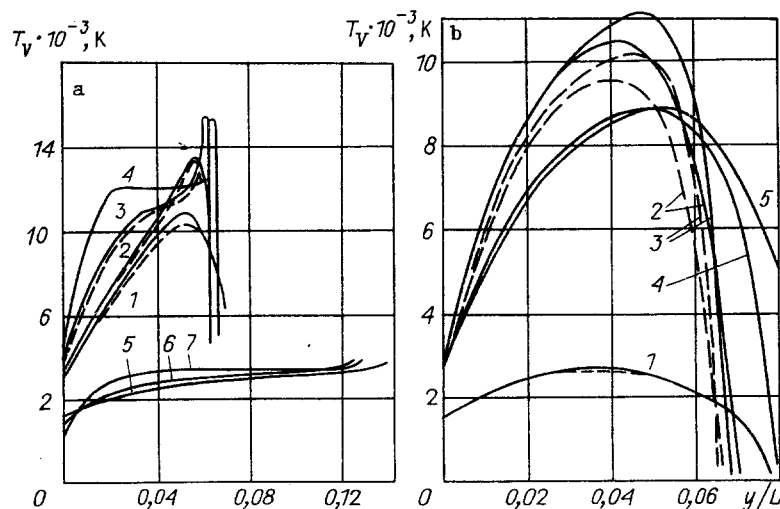


Fig. 3

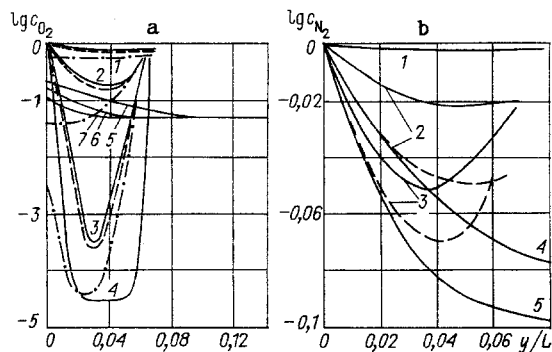


Fig. 4

This conclusion is confirmed also by the calculations carried out for nitrogen (regimes 5-7) and shown in Figs. 2b-4b (curves 1-3). The degree of dissociation of the  $N_2$  molecule is not high (Fig. 4b); however, the influence of the dual temperature function  $k_d(T, T_V)$  [the dashed lines represent calculation for the case of  $k_d^0(T)$ ] on the distribution of the parameters in the TVSL is considerable, because of the great activation energy.

Let us note that when  $Re_{0f} \leq 2.4$  the flow in the viscous shock layer is close to that in the "frozen-in" layer and the influence of the differences in the description of the dissociation rates on the structure of the flow is exceedingly small.

4. Simulation of the Streamlining of Bodies in an AT. The question as to the possibility of simulating the conditions of natural flight in an aerodynamic wind tunnel, where we have a considerable degree of dissociation in an unperturbed flow incident on the model, was examined in [3, 8, 16, 17].

The analysis conducted in [3, 17] demonstrated that under the conditions of the tunnel experiment, the flow incident on the model was vibrational and "frozen-in" and the degree of molecular dissociation is close to being "frozen in." According to estimates [3],  $c_{O_2} = 0.05$ ,  $T_{V,\infty} = 3880$  K. Parameters typical for the incident flow in an AT were selected:  $V_\infty = 2.5$  km/sec,  $\rho_\infty = 10^{-4}$  kg/m<sup>3</sup>, the characteristic linear scale of the model  $\ell = 0.011, 0.050, 0.186$  m for  $Re_{0f} = 7.08, 31.8, 118.7$  (regimes 1-3). The distributions of the mixture parameters across the TVSL can be found in Figs. 2a-4a (curves 5-7). The nature of the distribution of the molecular oxygen concentration (see Fig. 4a) near the catalytic surface of the sphere is significantly different between the tunnel and natural cases, which can be ascribed to the considerable differences in the course of the physicochemical processes; there is also a significant difference in the degree of vibrational molecular excitation (see Fig. 3a). Here the difference in  $C_h$  (see Table 1) is observed over the entire range of  $Re_{0f}$  and amounts to  $C_{h,At} \approx 0.6C_{h,H}$ . The latter circumstance can be ascribed to the fundamental differences in the states of the gas in front of the shock wave and to temperature factors. Processing the heat-transfer data through the introduction of the Stanton number  $St = q/[\rho_\infty V_\infty (H_0 - H_w)]$  [8, 17] significantly improves the comparability of the results obtained in the tunnel and natural experiments.

Let us point out yet another possibility of formulating and conducting the experiments in the AT in order to simulate the flow in a TVSL. We know [8, 17] that the scale of the chemical processes in a viscous shock layer in front of the body is primarily binary and the recombination reaction need not be taken into consideration. Within the context of this approximation, in [16] we find a study of the nonviscous nonequilibrium flow near the critical line of a blunted body, this study based on an analogy with the flows that arise behind the direct compression shocks. The developed approach is based on the assumption of a linear relationship between the enthalpy of the gas and the concentration of the chemical components, thus making it possible to establish the rules for recalculation of the experimental data in flows exhibiting an initially dissociated gas for the conditions of flight in an unperturbed atmosphere.

Assuming that the enthalpy of deceleration and the forward thrust in each of these compared cases are identical, it is not difficult to establish the following relationships between the velocities of the flows and their densities [16]:

$$\bar{V}_\infty = \sqrt{V_\infty^2 - 2c_{a,\infty} R_g T_{diss} / M_M}, \quad \bar{\rho}_\infty = \rho_\infty (V_\infty / \bar{V}_\infty)^2. \quad (4.1)$$

Here the overhead bar denotes quantities corresponding to the conditions within the AT;  $M_M$  represents the molecular weight of the molecules;  $R_g$  is the universal gas constant;  $T_{diss} = D/k$ . With a relatively small concentration of atoms in the incident flow ( $c_{a,\infty} \leq 0.3$  [16]) at a comparatively small distance behind the shock wave we achieve the required agreement for the flows being considered here. In this zone of inviscid flow the values for temperature, density, and concentration coincide with acceptable accuracy. In the light of (4.1) it becomes necessary to alter the form of the similarity criteria  $Re_{of} = \text{const}$  and  $\rho_\infty L = \text{const}$ , obtained in [17] under the condition that the velocities are constant. Analysis of the similarity criteria which were used in the aerothermodynamics of the rarefied gas was undertaken in [17-19]. Thus, from the analysis of the Boltzmann equations in [19] we obtained the similarity criterion first introduced by Cheng [13] in analyzing the structure of the thin viscous shock layer:

$$K = \frac{\rho_\infty V_\infty L}{\mu_*} \frac{R_g T_*}{M_M V_\infty^2} = \frac{\bar{\rho}_\infty \bar{V}_\infty l}{\bar{\mu}_*} \frac{R_g \bar{T}_*}{M_M \bar{V}_\infty^2}. \quad (4.2)$$

In an ideal gas ( $\gamma = \text{const}$ ), assuming that  $T_* = T_{of}$ , it is not difficult to obtain  $\varepsilon \approx (R_g/M_M) T_* / V_\infty^2$  and  $K = Re_{of} \varepsilon$ . Taking the gas deceleration temperature in the absence of dissociation  $\bar{T}_* = T_* = T_{of}$  as the characteristic temperature, we will have

$$K = \frac{\bar{\rho}_\infty \bar{V}_\infty l}{\mu_{of}} \frac{R_g T_{of}}{M_M \bar{V}_\infty^2}.$$

From relationship (4.2) it is easy to establish a relationship between the linear scale of the natural object  $L$  and of the model  $l$ :

$$l/L = (\bar{V}_\infty/V_\infty)^3. \quad (4.3)$$

Fulfillment of relationships (4.1) and (4.3) makes it possible to establish agreement between the flows of a viscous gas under natural flight conditions and for the tunnel experiment as functions of gas dissociation in the antechamber of the AT.

Specific calculations have been carried out for nitrogen, chosen as the working gas fluid in the AT because of its relatively low degree of molecular dissociation at the outlet from the nozzle. The distributions of  $T$ ,  $T_V$ , and  $c_{N_2}$  across the TVSL for an ideal catalytic surface are found in Figs. 2b-4b (curves 4, 5 correspond to simulation of regimes 6, 7). It was assumed that the flow incident on the model is dissociated ( $c_{N,\infty} = 0.2$ ). According to relationships (4.1)-(4.3),  $Re_{of,AT}$  must be smaller than  $Re_{of}$ , which leads to differences in the distributions of the parameters in the TVSL and to some excess in the flow of heat above the "natural" values of  $C_{h,AT} > C_h$  (see Table 1). However, processing of the heat-transfer data in the form of  $St(K)$  functions considerably improves correlation of the data obtained for natural flight conditions and for the conditions prevailing in the AT.

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A SELF-SIMILAR PROBLEM DEALING WITH THE ONE-DIMENSIONAL COLLISION  
OF TWO HALF SPACES OF A NONLINEAR-ELASTIC MATERIAL

I. E. Agapov, A. M. Belogortsev, A. A. Burenin,  
and A. V. Rezunov

UDC 539.3

The problems associated with investigating the processes of high-speed collisions in deformable bodies is of great theoretical and applied significance. However, the solution of these problems is associated with considerable difficulties, ascribed to the wave nature of strain propagation, as well as by the need to make use of nonlinear mechanical models. Self-similar problems from nonlinear elasticity theory provide specific information regarding the behavior of bodies under conditions of intensive dynamic load, and the solution for these problems can be found analytically or comparatively easily with numerical methods. The qualitative features of the process of propagating perturbations has been studied [1], and solutions have been obtained for a series of specific problems [2-4].

In the present study we examine a one-dimensional problem dealing with the collision of two nonlinear-elastic half spaces, one of which is addressed, and the second in motion at a constant translational velocity. At the boundary of interaction between the bodies it is assumed that the Coulomb dry-friction law is fulfilled. We examine the nature of the deformation, and we present results from a numerical study of the problem, as well as a comparison with the solution for the linear case.

The Ox<sub>1</sub> axis in the Cartesian coordinate system is directed perpendicular to the half-space boundary (Fig. 1). At the initial instant of time the half space x<sub>1</sub> > 0 is fixed,

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